

Disappearance of equilibrium with thermal spin bath induced by non-Markov process

Zhihai Wang,¹ Yu Guo,^{1,2} and D. L. Zhou¹

¹*Beijing National Laboratory for condensed matter physics,
Institute of Physics, Chinese Academy of Sciences, Beijing, 100190, China*

²*School of Physics and Electronic Science, Changsha University of Science and Technology, Changsha 410114, China*

In most cases, a small system weakly interacting with a thermal bath will finally reach the thermal state with the temperature of the bath. We show that this intuitive picture is not always true by a spin star model where the non-Markov effect predominates in the whole dynamical process. The spin star system consists of a central spin homogeneously interacting with an ensemble of identical noninteracting spins. We find that the correlation time of the bath is infinite, which implies that the bath has a perfect memory, and that the dynamical evolution of the central spin must be non-Markovian. A direct consequence is that the final state of the central spin is not the thermal state equilibrium with the bath, but a steady state which depends on its initial state.

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I. INTRODUCTION

A quantum system can never completely be isolated from its surrounding environment, and in many cases it must be treated as open system [1]. The investigations on quantum open systems have attracted more and more attentions and many relative theoretical approaches have been developed, such as the Markov approximation [1–3], the quantum jump approach [4, 5], the quantum state diffusion [6, 7], the canonical transformation [8, 9] as well as the path integral method [10].

It is a common sense that, when a small system interacts with a large thermal bath, it will finally reach the thermal state in equilibrium with the bath. However, we will show that it is not always true by studying the reduced dynamics of the central spin which homogeneously interacts with an ensemble of identical non-interacting spins. Such spin star configuration [11–20] can be realized in many solid-state quantum systems, for example, the semiconductor quantum dot [21–26] or the nitrogen-vacancy center in diamond [27, 28].

Generally speaking, the environment interacting with the small system is composed of multimode bosons [10] or spins [29, 30]. For such an environment, the correlation time of the environment is much shorter than the characteristic time of the system. Especially, under the Markov approximation, the correlation function is assumed to be a δ type of time interval [1–3], which implies that the environment's memory effect can be neglected during the time evolution. However, in our spin star configuration, the correlation function of the bath is found to be a periodic oscillation function without damping. The thermal bath then has a perfect memory and the reduced dynamics of the central spin must be non-Markovian. As a result, the central spin can not reach the thermal state in equilibrium with the bath.

In this paper, we study the dissipation of the central spin in a spin star system. Compared to the models proposed in Refs. [11, 12], the free Hamiltonians of the central spin and the thermal bath, which do not commute

with the interaction Hamiltonian, are taken into account. The dynamics of the central spin is then dramatically different from that without the free terms. In our system, the whole Hilbert space can be decomposed into a direct sum of smaller subspaces due to the exchange symmetry of the bath spins, and then we are able to study the dynamics of the system exactly. We find that the final state of the central spin is a steady state which depends on its initial state, instead of the thermal equilibrium state as obtained under the Markov approximation [31]. The initial thermal equilibrium bath state is composed of a minority of the eigenstates, and most of the bath states remain unchanged during the time evolution, which leads to a perfect memory of the bath. This explanation is confirmed by several distinct discrete peaks in the spectrum diagram.

The paper is organized as follows. In Sec. II, we set up our model as a spin star configuration and investigate the reduced dynamics of the central spin in detail. In Sec. III, we find that the central spin can not reach the thermal state in equilibrium with the bath, and we further analyze the underlying physical mechanism with the help of fast Fourier transformation. In Sec. IV, we demonstrate that the traditional Markov approximation is not applicable by investigating the correlation time of the thermal bath as well as the mutual entropy between the central spin and the thermal bath. In Sec. V, we draw the conclusions and make several remarks.

II. THE MODEL AND THE REDUCED DYNAMICS OF THE CENTRAL SPIN

A. The Hamiltonian

We consider a spin star configuration as shown in Fig. 1. The system is composed of $N + 1$ localized spin $1/2$ particles. The central spin is immersed in a bath of N spins which are arranged in a circle.

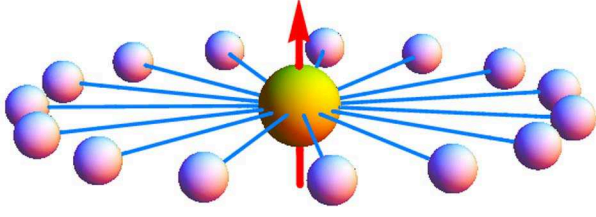


FIG. 1. (Color online) A schematic diagram of spin star configuration. The central spin with the energy level spacing ω_0 homogeneously couples to the surrounding bath spins with the energy level spacing ω . The bath spins do not interact with each other.

The Hamiltonian of the global system is

$$H = \frac{\omega_0}{2} \sigma_z^{(0)} + \frac{\omega}{2} \sum_{i=1}^N \sigma_z^{(i)} + g \sigma_x^{(0)} \sum_{i=1}^N \sigma_x^{(i)}, \quad (1)$$

where σ_z and σ_x are the Pauli operators, ω_0 and ω are the frequencies of the central spin and the bath spins respectively. The first two terms in the r.h.s. of Eq. (1) are the Hamiltonians for the central spin and the bath spins, the last term describes that the central spin homogeneously couples to the bath spins with the coupling strength g . Throughout this paper, we set $\hbar = 1$.

Since the Hamiltonian (1) is invariant with respect to the exchange of arbitrary two spins in the bath, it is reasonable to define the collective operators:

$$J_z = \sum_{i=1}^N \frac{\sigma_z^{(i)}}{2}, J_x = \sum_{i=1}^N \frac{\sigma_x^{(i)}}{2}. \quad (2)$$

Then the Hamiltonian (1) becomes

$$H = \frac{\omega_0}{2} \sigma_z^{(0)} + \omega J_z + 2g \sigma_x^{(0)} J_x. \quad (3)$$

The central spin thus couples to a collective bath angular momentum. The angular momentum of the bath is a changeable one and the total angular momentum j can take values as $j = N/2, N/2 - 1, \dots, N/2 - [N/2]$, where the notation $[a]$ denotes the maximum integer not larger than a . It is obvious that the total angular momentum of the thermal bath is a conserved quantity, i.e., $[J, H] = 0$.

Utilizing the exchange symmetry of the bath, we investigate the reduced dynamics of the central spin under the influence of the thermal bath. The state of the central spin is described by the reduced density matrix

$$\rho^S(t) = \text{Tr}_B[\rho^{SB}(t)], \quad (4)$$

where Tr_B is the partial trace over the degree of freedom of the thermal bath and $\rho^{SB}(t)$ is the density matrix of the global system including the central spin and the thermal bath at time t . The density matrix $\rho^{SB}(t)$ undergoes the unitary evolution

$$\rho^{SB}(t) = \exp(-iHt) \rho^{SB}(0) \exp(iHt). \quad (5)$$

In our consideration, the reduced density matrix of the central spin ρ^S is a 2×2 matrix in the basis of states $\{|\uparrow\rangle, |\downarrow\rangle\}$ with $|\uparrow\rangle$ and $|\downarrow\rangle$ being the spin up and spin down states respectively. The diagonal element $\langle \uparrow | \rho^S | \uparrow \rangle$ corresponds to the probability in its spin up state. In what follows, we will focus on the evolution of the probability and investigate the underlying physical mechanism.

B. The initial state of the thermal bath

We assume that the initial state of the global system can be factorized into an uncorrelated product state:

$$\rho^{SB}(0) = \rho^S(0) \otimes \rho^B(0) \quad (6)$$

with $\rho^S(0)$ and $\rho^B(0)$ being the initial density matrices for the central spin and the thermal bath respectively.

The initial state of the bath is a thermal equilibrium state

$$\rho^B(0) = \frac{\exp(-\beta \omega J_z)}{\text{Tr}[\exp(-\beta \omega J_z)]}, \quad (7)$$

where the z -component of the collective angular momentum J_z is defined in Eq. (2), and $\beta = 1/k_B T$ is the inverse temperature of the bath, with k_B being the Boltzmann constant, which will be set to unit in the following.

The Hilbert space of the bath is an N -fold tensor product of two dimensional spaces. A direct diagonalization of the Hamiltonian is practically impossible when the number of bath spins N is large. Fortunately, the huge Hilbert space \mathbb{C}^B can be decomposed into a direct sum of smaller invariant subspaces $\mathbb{C}_{j\gamma_j}^B$, i.e.,

$$\mathbb{C}^B = \oplus_{j\gamma_j} \mathbb{C}_{j\gamma_j}^B. \quad (8)$$

Here, the $2j+1$ dimensional subspace $\mathbb{C}_{j\gamma_j}^B$ is spanned by the states with the total angular momentum j ($j \leq N/2$). We introduce the index $\gamma_j = 1, 2, \dots, \alpha_j^N$ to label the different invariant subspaces with the same j , where the degeneracy is induced by the different possible orientations of the spins in the thermal bath. A simple calculation gives the degeneracy

$$\alpha_j^N = \binom{N}{\frac{N}{2} - j} - \binom{N}{\frac{N}{2} - j - 1}. \quad (9)$$

The above analysis naturally motivates us to rewrite the initial density matrix of the thermal bath $\rho^B(0)$ in the direct summation of the invariant subspaces. To this end, we define a set of orthogonal bases $|\gamma_j, j, m\rangle$ in the subspace $\mathbb{C}_{j\gamma_j}^B$. These states are the eigenstates of J^2 with eigenvalues $j(j+1)$ and of J_z with eigenvalues m . Then,

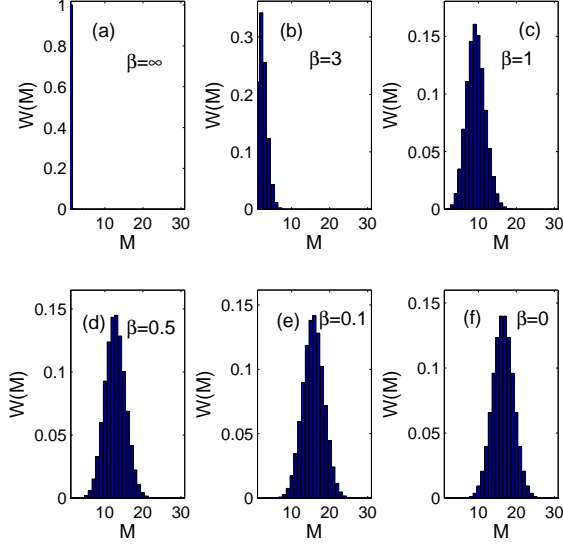


FIG. 2. (Color online) The relative weight $W(M)$ versus the excitation numbers M in different temperatures. The number of bath spins is $N = 31$. We use $\beta = 1/k_B T$ to represent the temperature of the bath. The corresponding temperature gradually increases from (a) to (f). All parameters are in the unit of ω .

$\rho^B(0)$ can be written as

$$\begin{aligned} \rho^B(0) &= \frac{1}{Z} \exp(-\beta\omega J_z) \\ &= \frac{1}{Z} \sum_j \sum_{m=-j}^j \sum_{\gamma_j=\alpha_j^N}^{\gamma_j=\alpha_j^N} \exp(-\beta\omega m) |\gamma_j, j, m\rangle \langle \gamma_j, j, m|, \end{aligned} \quad (10)$$

where the partition function Z is defined as

$$Z \equiv \text{Tr} \left[\sum_j \sum_{m=-j}^j \sum_{\gamma_j=\alpha_j^N}^{\gamma_j=\alpha_j^N} \exp(-\beta\omega m) |\gamma_j, j, m\rangle \langle \gamma_j, j, m| \right]. \quad (11)$$

For the thermal equilibrium state given by Eq. (10), the relative weight for M excitations among N bath spins (i.e., M spins in their up states and the other $N-M$ spins in their down states) in the thermal bath is

$$W(M) = \binom{N}{M} \frac{\exp[-\beta\omega(M - N/2)]}{Z}. \quad (12)$$

In Fig. 2, we plot the relative weight $W(M)$ ($N = 31$) in different temperatures. It shows that, the states with low excitations play a predominate role in the low temperatures. As the rising of the temperature, the states with high excitations gradually emerge in the initial state, and the weights of states with low excitations are suppressed. When the temperature is high enough, the distribution is temperature independent (shown in Fig. 2 (e) and (f)).

We point out that, the excitation number is not a conserved quantity in our system. Therefore, the states with different excitations may couple to each other, generally depends on whether they belong to the same invariant subspace $\mathbb{C}_{j\gamma_j}^B$.

C. The reduced dynamics of the central spin

In this subsection, we will study the reduced dynamics of the central spin under the influence of the thermal bath. We illustrate the dissipation of the central spin by calculating the probability in its spin up state.

As discussed in the previous subsection, the Hilbert space of the thermal bath is composed by a direct sum of smaller invariant subspaces. Furthermore, the Hilbert space \mathbb{C}^{SB} of the global system (including the central spin and the thermal bath) is given by $\mathbb{C}^{SB} = \oplus_{j\gamma_j} \mathbb{C}_{j\gamma_j}^{SB}$, where

$$\mathbb{C}_{j\gamma_j}^{SB} = \mathbb{C}^S \otimes \mathbb{C}_{j\gamma_j}^B \quad (13)$$

with $\mathbb{C}_{j\gamma_j}^B$ being defined in Eq. (8) and \mathbb{C}^S being the 2-dimensional Hilbert space for the central spin. In the basis of $|\mu; \gamma_j, j, m\rangle \equiv |\mu\rangle \otimes |\gamma_j, j, m\rangle$, the Hamiltonian is written as $H = \sum_j \sum_{\gamma_j=1}^{\alpha_j^N} \mathcal{H}_{j\gamma_j}$ with

$$\mathcal{H}_{j\gamma_j} = \sum_{\mu, \mu'=\downarrow, \uparrow} \sum_{m, m'=-j}^j H_{\mu, m}^{\mu', m'} |\mu'; \gamma_j, j, m'\rangle \langle \mu; \gamma_j, j, m| \quad (14)$$

where the matrix element $H_{\mu, m}^{\mu', m'}$ is defined as

$$H_{\mu, m}^{\mu', m'} \equiv \langle \mu'; \gamma_j, j, m' | H | \mu; \gamma_j, j, m \rangle. \quad (15)$$

Correspondingly, we rewrite the initial density matrix of the global system as $\rho^{SB}(0) = \sum_j \sum_{\gamma_j=1}^{\alpha_j^N} Z_{j\gamma_j} \rho_{j\gamma_j}^{SB}(0) / Z$ with

$$\rho_{j\gamma_j}^{SB}(0) = \frac{1}{Z_{j\gamma_j}} \rho^S(0) \otimes \sum_{m=-j}^j \exp(-\beta\omega m) |\gamma_j, j, m\rangle \langle \gamma_j, j, m| \quad (16)$$

where

$$Z_{j\gamma_j} = \text{Tr} \left[\sum_{m=-j}^j \exp(-\beta\omega m) |\gamma_j, j, m\rangle \langle \gamma_j, j, m| \right] \quad (17)$$

is introduced to guarantee the unity trace of $\rho_{j\gamma_j}^{SB}(0)$. In the above equations, $\mathcal{H}_{j\gamma_j}$ and $\rho_{j\gamma_j}^{SB}(0)$ can be understood as the projection of the Hamiltonian and the initial density matrix on the subspace $\mathbb{C}_{j\gamma_j}^{SB}$ respectively.

In the subspace $\mathbb{C}_{j\gamma_j}^{SB}$, the “density matrix” $\rho_{j\gamma_j}^{SB}$ experiences the unitary evolution

$$\rho_{j\gamma_j}^{SB}(t) = \exp(-i\mathcal{H}_{j\gamma_j}t) \rho_{j\gamma_j}^{SB}(0) \exp(i\mathcal{H}_{j\gamma_j}t). \quad (18)$$

and the reduced density matrix for the central spin is $\rho_{j\gamma_j}^S(t) = \text{Tr}_{Bj\gamma_j}[\rho_{j\gamma_j}^{SB}(t)]$, where $\text{Tr}_{Bj\gamma_j}[\cdot]$ represents the partial trace over the degree of freedom of the thermal bath in the subspace $\mathbb{C}_{j\gamma_j}^{SB}$.

Hence the probability for the central spin in its spin up state is

$$P(t) = \frac{1}{Z} \sum_j \alpha_j^N Z_{j\gamma_j} \langle \uparrow | \rho_{j\gamma_j}^S(t) | \uparrow \rangle. \quad (19)$$

III. RESULTS AND DISCUSSIONS

In general cases, a small system interacting with a large thermal bath will be thermalized. In other words, the small system will reach the thermal state in equilibrium with the bath. For example, for a single spin-1/2 particle as the small system which is described by the Hamiltonian $H = \omega_0 \sigma_z/2$, the final density matrix would be

$$\rho(t = \infty) = \frac{\exp(-\beta\omega_0\sigma_z/2)}{\text{Tr}[\exp(-\beta\omega_0\sigma_z/2)]} \quad (20)$$

and then the probability in its up state is

$$P(t = \infty) = \frac{1}{2} - \frac{\tanh(\beta\omega_0/2)}{2}. \quad (21)$$

It shows that the probability depends only on the temperature of the thermal bath but not on the initial state of the spin. Such results can be obtained under the Markov approximation [31, 32]. In the following, however, we will show that this intuitive picture is not true in our system.

Through a straightforward numerical calculation based on Eq. (19), we illustrate the dynamics of the probability for the central spin in its spin up state in Fig. 3. In the figure, the red (blue) curve represents the case when the central spin is in its up (down) state initially. It shows that, the final state of the central spin depends on its initial state. As shown in the figure, the probability oscillates around a fixed value and the amplitudes of the oscillation decrease with the increase of the bath temperature. In other words, the probability tends to be a constant when the temperature of the spin bath is high enough.

Furthermore, when the central spin initially prepared in the spin up or spin down state, we will have $\langle \sigma_x^{(0)} \rangle = \langle \sigma_y^{(0)} \rangle = 0$ during the evolution. This can be clarified in the viewpoint of Z_2 symmetry of the system. By the analogy of Ref. [33], we define the parity operator $\mathcal{P} \equiv \exp[i\pi(J_z + \sigma_z^{(0)}/2)]$ which is conserved with respect to the Hamiltonian (3), namely, $[\mathcal{P}, H] = 0$. In our consideration, the initial state has a fixed parity and it will stay in the same parity during time evolution due to the conservation of the parity. However, the central spin operators $\sigma_x^{(0)}$ and $\sigma_y^{(0)}$ change signs under the transformation of the parity, i.e., $\mathcal{P}^\dagger \sigma_x^{(0)} \mathcal{P} = -\sigma_x^{(0)}$, $\mathcal{P}^\dagger \sigma_y^{(0)} \mathcal{P} = -\sigma_y^{(0)}$.

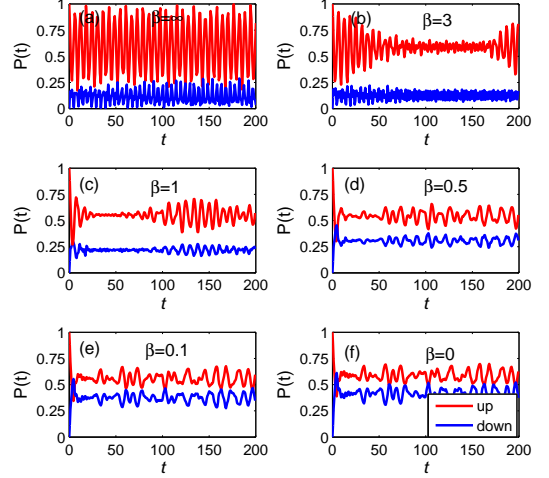


FIG. 3. (Color online) The probability for the central spin in its spin up state as a function of time t . The initial state of the central spin is spin up state for red lines and spin down state for blue lines. The parameters are set as the $\omega_0 = 1, g = 0.1$. The inverse of temperature is (a) $\beta = \infty$, (b) $\beta = 3$, (c) $\beta = 1$, (d) $\beta = 0.5$, (e) $\beta = 0.1$, (f) $\beta = 0$. The number of spins in the thermal bath is $N = 31$. All parameters are in the unit of ω .

Therefore, their average values will keep zero at any time t .

The above discussions show that, when the thermal bath is large enough, the central spin will evolve into a state which satisfies $\langle \sigma_x^{(0)} \rangle = \langle \sigma_y^{(0)} \rangle = 0$ and $\langle \sigma_z^{(0)} \rangle \approx \text{constant}$. Therefore, we can safely conclude that the central spin will reach a steady state instead of the thermal equilibrium state.

As demonstrated above, the central spin can not be perfectly thermalized as expected by interacting with an ensemble of identical spins. The main reason is that, the initial thermal state of the bath is composed of a minority eigen states of the Hamiltonian, and therefore the energy level transitions during time evolution only occur among the few eigen states which are related to the initial state, leaving other states unchanged. As a result, the bath has a perfect memory and the initial thermal equilibrium is broken as the time elapses, which leads to the disappearance of the equilibrium state of the central spin.

To investigate the energy level transition in detail, we switch, by the fast Fourier transformation (FFT) technology, from the time domain to the frequency domain, where we can obtain the transition frequency quantitatively.

We plot the frequency spectrum diagram for the probability obtained in Eq. (19) in different temperatures in Fig. 4 assuming that the central spin is in its spin up state initially. It follows from Fig. 4 that, there exist two distinct peaks in zero temperature ($\beta = \infty$). As the temperature rises, the high frequency peak gradually de-

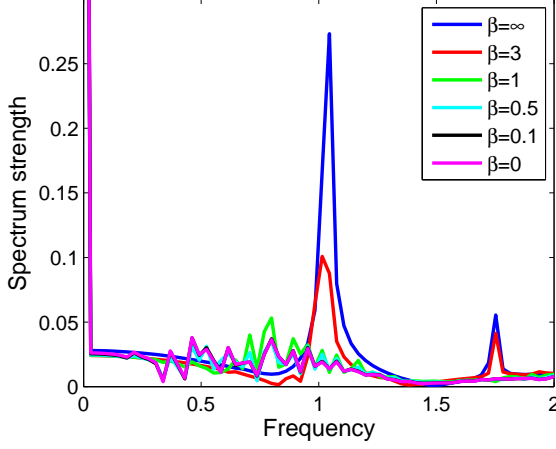


FIG. 4. (Color online) The frequency spectrum of the probability in Eq. (19) by FFT. The parameters are same as that in Fig. 3. The central spin is in the spin up state initially.

creases until disappears. Meanwhile, more and more low frequency peaks with relative weak strength emerge. Actually, all of these peaks correspond to the related energy level transitions.

In the low temperatures, nearly all of the bath spins are in their spin down states initially. The initial state only locates in the subspaces $\mathbb{C}_{j\gamma_j}^B$ with large j , for example, $j = N/2, N/2 - 1, N/2 - 2$. Moreover, in the case of weak coupling, the transitions only occur among a few lowest intra-subspace energy levels. This is similar to the Rabi model, which describes the interaction between a two-level system and a single mode bosonic field [34]. In zero temperature, the Rabi oscillation like phenomena recurs in Fig. 3(a) where we note that the “anti-rotating wave” term leads to the inequality of the oscillation amplitudes. As for the case of the low temperatures, it also reproduces the phenomena of collapse and revival, which can be observed clearly in Fig. 3(b).

As the temperature further rises, the components of initial state emerge in more and more subspaces $\mathbb{C}_{j\gamma_j}^B$ with small j and the relative weights in the subspaces with large j gradually decrease (shown in Fig. 4). In these small j subspaces, the energy level differences are smaller than those in the large j subspaces. Therefore, the peaks with high frequency gradually disappear and the low frequency peaks arise (shown in Fig. 4). When the temperature is high enough (for example when $\beta < 0.1$), the frequency spectrum and the dynamical evolution process are temperature independent. We see that the curves for $\beta = 0.1$ and $\beta = 0$ coincide with each other in Fig. 4 and Figs. 3(e, f).

The frequency spectrum in Fig. 4 shows that the probability (the r.h.s. of Eq. (19)) is a superposition of oscillation functions with different frequencies and all of the components evolve with time in unison during a short time. However, the interfere effect among different com-

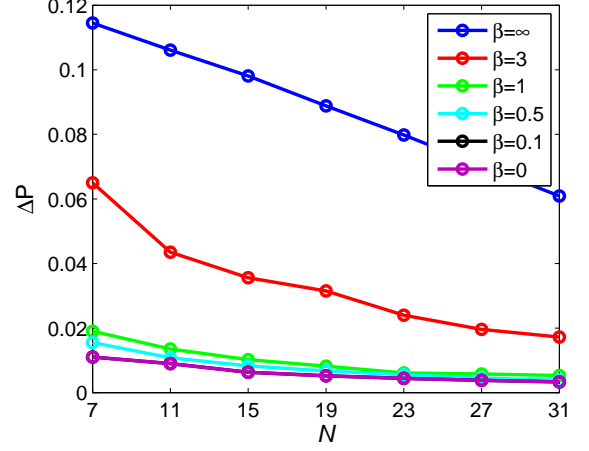


FIG. 5. (Color online) The fluctuation of the probability ΔP as a function of the bath spins' number N for different temperature. The parameters and initial state are same as that in Fig. 4. The case of $\beta = 0.1$ and $\beta = 0$ completely coincide with each other.

ponents becomes important once the time is long enough, and the probability oscillates around a fixed value as the time elapse. The different behaviors of the probability in two time scales are clearly shown in high temperatures in Fig. 3 (d-f).

To depict the quantum interference phenomena quantitatively, we define the probability fluctuation

$$\Delta P = \overline{(P(t) - \bar{P})^2}, \quad (22)$$

where \bar{a} denotes the mean value of a over time. In Fig. 5, we plot the curve of ΔP as a function of number of bath spins in different temperatures. It shows that ΔP decreases monotonously as the increase of the spins number and the temperature of the bath. Both of the increasings imply that more components participate the summation in Eq. (19), it is the destructive interference of the different components that weakens down the fluctuations.

IV. THE INVALIDITY OF THE MARKOV APPROXIMATION

In the study of the open system, the Markov approximation is widely used to discuss the reduced dynamics of the system. The Markov approximation usually works well when the small system weakly couples to a large thermal bath. Under the Markov approximation, the typical time of the internal correlation of the bath is assumed to be much shorter than the characteristic relaxation time of the system, which implies that the bath correlation decays more rapidly compared to the system damping. As a result, the bath loses its memory and does not change significantly with time. Therefore, the system does not affect the bath significantly, but the bath does

affect the system. However, the Markov approximation loses its effectiveness in our system even if the conditions of large thermal bath and weak coupling are both satisfied. In this section we will show the invalidity of the Markov approximation in two aspects: the internal correlation time of the bath tends to be infinite and the central spin does correlate with the bath all the time during the time evolution.

Firstly, we study the correlation of the thermal bath. It is convenient to work in the interaction picture with $H_0 = \omega_0 \sigma_z^{(0)}/2 + \omega \sum_i^N \sigma_z^{(i)}/2$, the internal correlation function of the thermal bath is calculated as

$$\langle \Gamma^\dagger(t') \Gamma(t) \rangle_B = g^2 \frac{N \exp(-\beta\omega/2)}{2 \cosh(\beta\omega/2)} e^{-i\omega(t-t')/2} \quad (23)$$

with $\Gamma(t) = g \sum_j \sigma_-^{(j)} e^{-i\omega_j t}$ and $\langle \cdot \rangle_B$ being the average over the thermal equilibrium state of the bath in temperature T which is described by the density matrix

$$\rho^B = \frac{\exp\left(-\beta\omega \sum_i \sigma_z^{(i)}/2\right)}{\text{Tr} \left[\exp\left(-\beta\omega/2 \sum_i \sigma_z^{(i)}/2\right) \right]}. \quad (24)$$

Here, Tr means the trace over the degree of freedom of the spin bath.

It is shown that the correlation function in Eq. (23) is a periodic oscillation function without any damping. Therefore, the memory effect of the thermal bath can not be neglected and the condition of the Markov approximation is naturally broken. The same conclusion in spin star system is also obtained by a direct analysis of the Redfield master equation in Ref. [11].

Secondly, we will discuss the correlation between the central spin and the thermal bath applying the concept of the mutual entropy. In quantum information, the mutual entropy between two subsystems (the central spin and the thermal bath in our system) is defined as:

$$\mathcal{I} = S(\rho^S) + S(\rho^B) - S(\rho^{SB}) \quad (25)$$

where $S = -\sum_i \lambda_i \log_2 \lambda_i$ (λ_i being the eigenvalues of ρ) is the von Neumann entropy [35, 36]. Under the Markov approximation, the influence of the open system to the bath is ignored, and the global system will stay at the tensor product state

$$\rho^{SB}(t) = \rho^S(t) \otimes \rho_{th}^B \quad (26)$$

with ρ_{th}^B being the initial thermal equilibrium state of the bath. The mutual entropy of such product state is zero because the correlation between two subsystems disappears. However, this is not the case for our system. In Fig. 6, we plot the mutual entropy obtained by our approach in different temperatures. It shows that the mutual entropy never disappears, which implies that the central spin does correlate with the thermal bath, and is incompatible with the result from the Markov approximation. This also verifies the invalidity of the Markov approximation.

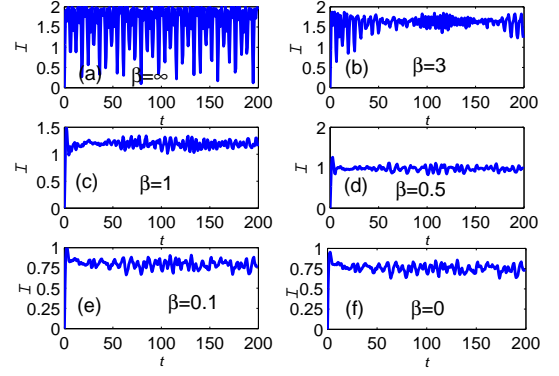


FIG. 6. (Color online) The mutual entropy between the central spin and the thermal bath as a function of time at different temperatures. The parameters and the initial state are same as that in Fig. 3.

V. CONCLUSIONS AND REMARKS

In this paper, we have studied the reduced dynamics of the central spin in a spin star system and calculated the probability of the central spin in its spin up state. We find that the probability oscillates around a fixed value and the oscillation amplitudes decrease with the increase of temperature and the number of bath spins. To investigate the energy level transitions during the time evolution, we utilize the FFT technique to obtain the spectrum diagram. In the low temperatures, there only exist two distinct peaks and the probability function shows periodicity which is similar to the Rabi oscillation. As the increase of the temperature, it exhibits more peaks in the spectrum diagram, which shows that the probability function is composed of finite components with different frequencies. It is the destructive interfere of different compositions that weakens down the oscillation.

Here we emphasize that the reduced state of the central spin can be constructed from its probability in the up state. This is because the average values of the x -component and y -component keep zero during the time evolution, which is directly proved from a consideration of parity conservation.

In our system, the conventional Markov approximation is not reasonable even in the case of weak system-bath coupling. On one hand, it is discovered that the correlation time of the bath tends to be infinite, which implies that the bath has a perfect memory. On the other hand, the correlation between the central spin and the thermal bath also implies the invalidity of the Markov approximation in our system. Due to the non-Markov effect, the central spin can not be perfectly thermalized by interacting with a bath of identical spins. Instead, it will evolve into a steady state which depends on its initial state.

Technically, we point out that the Hilbert space is too huge to make a direct numerical calculation in our system when the number of bath spins is very large. However,

the huge Hilbert space can be decomposed into a direct sum of smaller subspaces utilizing the exchange symmetry of the bath spins and we are able to study the dynamics of the system exactly. By using of the same approach, the entanglement and other problems in quantum information related to spin star system can be solved without any approximation or specific numerical technique.

In summary, we give a spin star system to demonstrate a quantum open system without the Markov approximation for whatever weak coupling between the system and the environment. In our system, a considerable correlation between the system and the environment exists even in a long time, and the central spin will finally evolve into a steady state dependent of its initial state. We

hope that our investigation on this simple system will increase our understandings on the dynamics of quantum open system, a central topic in the physical realizations of quantum information processes.

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